10/23 (Day 54)

what you need to know
from yesterday....

Dot product:

$$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y$$

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$$

θ = angle in between $\vec{u}$ & $\vec{v}$

① $\vec{u} \cdot \vec{v} = 0 \Rightarrow \theta = 90^\circ$

vectors are perpendicular

② $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \Rightarrow \theta = 0^\circ$

vectors have same direction

example :

$$\vec{u} = 2\vec{i} + 3\vec{j}$$
$$\vec{v} = -2\vec{i} + 5\vec{j}$$

Find $\vec{u} \cdot \vec{v} = (2)(-2) + (3)(5)$

$= -4 + 15$

$= 11$

Find θ, the angle between the 2 vectors...

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$$

$$11 = \left(\sqrt{2^2+3^2}\right)\left(\sqrt{4+25}\right)\cos\theta$$

$$11 = \left(\sqrt{13}\right)\left(\sqrt{29}\right)\cos\theta$$

$$\frac{11}{\sqrt{13}\sqrt{29}} = \cos\theta$$

$$\cos^{-1}\left(\frac{11}{\sqrt{377}}\right) = \theta$$

$$55.5^\circ = \theta$$

p. 471

#1, 5, 6, 7, 9, 11

Test on Tue.

Review

Law sines/cosines
Trig. identities
Trig. equations

"New"

Vectors
Word problems
Trig. double angle, even/odd, other identities

Trig. Identities

Show $\sin x(\cot x + \cos x \tan x) = \cos x + \sin^2 x$

$\sin x \cot x + \sin x \cos x \tan x$

$\cancel{\sin x}\,\frac{\cos x}{\cancel{\sin x}} + \sin x \cancel{\cos x}\,\frac{\sin x}{\cancel{\cos x}}$

$\cos x + \sin^2 x$

$\therefore$

QED

Trig equations

$[0, 360^\circ]$

$$2\sin x - 1 = 0$$

$$2\sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = \arcsin\left(\frac{1}{2}\right)$$

$$x = \sin^{-1}\left(\frac{1}{2}\right) + 360^\circ n$$

$$x = 180^\circ - \sin^{-1}\left(\frac{1}{2}\right) + 360^\circ n$$

$$x = 30^\circ, 150^\circ$$

$[0, 360^\circ]$

$[-360, 720^\circ]$

1. $3\tan^2 x - 1 = 0$

2. $2\sin(3x) - 1 = 0$

3. $\sec^2 x - 2\tan x = 4$

$x \in [-360, 720]$

---

4. $\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$

5. $\dfrac{\sec^2\theta - \tan^2\theta}{2\sin^2\theta + 2\cos^2\theta} = \dfrac{1}{2}$

6. $\dfrac{1}{1-\cos x} + \dfrac{1}{1+\cos x} = 2\csc^2 x$