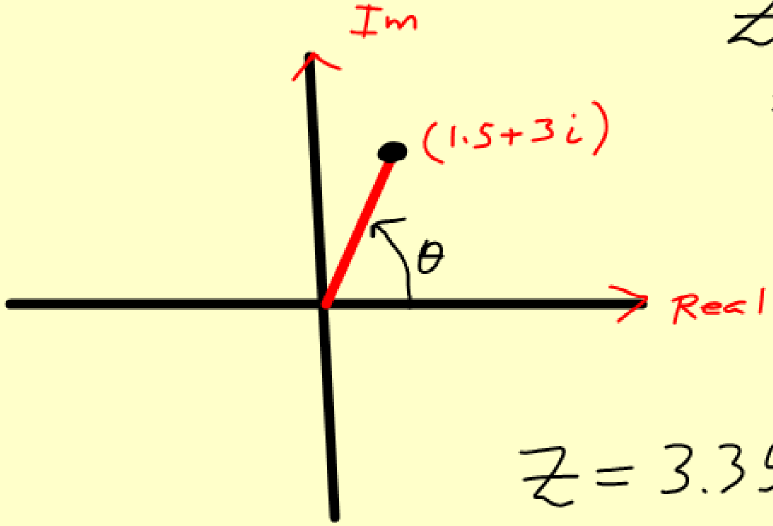
Day 64 -- Warm Up -- 11/9

Plot the complex number
z = (1.5 + 3i) and convert z into trig form



$$z = |z|\left[\cos\theta + i\sin\theta\right]$$

$$\Rightarrow |z| = r = \sqrt{(1.5)^2 + (3)^2}$$

$$\approx 3.35$$

$$\theta = \tan^{-1}\left(\frac{3}{1.5}\right)$$

$$= \tan^{-1}(2) = 63.4^\circ$$

$$z = 3.35\left(\cos(63.4^\circ) + i\sin(63.4^\circ)\right)$$

$$= 3.35 \text{ cis}(63.4^\circ)$$

$$i = \sqrt{-1}$$

$$z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$$

$$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$$

$$*\; z_1 z_2 = r_1 r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right]$$

$$z_1 z_2$$

$$= r_1(\cos\theta_1 + i\sin\theta_1)\, r_2(\cos\theta_2 + i\sin\theta_2)$$

$$= r_1 r_2 (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$$

$$= r_1 r_2 \left[\cos\theta_1\cos\theta_2 + i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2 + i^2\sin\theta_1\sin\theta_2\right]$$

$$= r_1 r_2 \left[\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + i(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2)\right]$$

$$= r_1 r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$$

example: $z_1 = 25\sqrt{2}\left(\cos -\frac{\pi}{4} + i \sin -\frac{\pi}{4}\right)$

$z_2 = 14\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$

Find: $z_1 z_2$ in trig and rect. forms

$$z_1 z_2 = 25\sqrt{2}(14)\left[\cos\left(-\frac{\pi}{4} + \frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{4} + \frac{\pi}{3}\right)\right]$$

$$z_1 z_2 = 350\sqrt{2}\left[\cos\left(\frac{\pi}{12}\right) + i \sin\left(\frac{\pi}{12}\right)\right]$$

$$= 478.12 + 128.2\, i$$

$$z^2$$

$$z = r(\cos\theta + i\sin\theta)$$

$$z^2 = r \cdot r\,(\cos(\theta+\theta) + i\sin(\theta+\theta))$$

$$z^2 = r^2[\cos(2\theta) + i\sin(2\theta)]$$

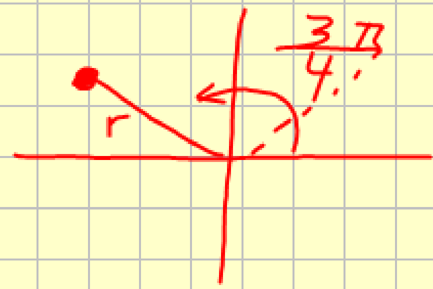
$$z^3 = zz^2 = rr^2[\cos(\theta+2\theta) + i\sin(\theta+2\theta)]$$

$$= r^3[\cos(3\theta) + i\sin(3\theta)]$$

$$z^n = r^n\left[\cos(n\theta) + i\sin(n\theta)\right]$$

De Movire Theorem

$$\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^8$$



$$\left[1\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)\right]^8$$

$$= (1)^8\left[\cos\left(\frac{8\cdot 3\pi}{4}\right) + i\sin\left(\frac{8\cdot 3\pi}{4}\right)\right]$$

$$r = \sqrt{\left(-\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2}$$

$$= \sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{1}$$

$$= 1$$

$$z^n = r^n\left[\cos(n\theta) + i\sin(n\theta)\right]$$

$$= \cos(6\pi) + i\sin(6\pi)$$

$$= 1 + i(0) = 1$$

Find

$$\left(1+i\sqrt{3}\right)^3$$

$$r = \sqrt{1^2+(\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) \approx 60^\circ \quad \text{rad}: \ \frac{\pi}{3}$$

$$r^3\left[\cos(3\theta) + i\sin(3\theta)\right]$$

$$(2)^3\left[\cos(\pi) + i\sin(\pi)\right]$$

$$8\left[-1 + i\,0\right] = -8$$

HW WS 6.6

#1 – 37 e.o.o.

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