Exploration 14-3a

1. 8, 11, 14, 17, 20, 23
2. 8 + 11 + · · · + 23 = 93
3. Student program. See the Programs for Graphing Calculators section of the *Instructor's Resource Book* for a sample program. The program is also available on the *Instructor's Resource CD* and at *www.keymath.com/precalc*.
4. 15,650
5. $t_n = n^2 + 1$
   $S_5 = 60 = 2 + 5 + 10 + 17 + 26$
6. 42,975
7. 1000, 1000(1.06) = 1060, 1060(1.06) = 1123.6, (1123.6)(1.06) = 1191.016;
   1000 + 1060 + 1123.6 + 1191.016 = 4374.616
8. $t_n = 1000(1.06)^{n-1}$
   The program gives the same answer.
9. 79,058.1862…
10. $t_n = 800(0.9)^{n-1}$
    $S_{10} = 5210.5724\ldots$, $S_{20} = 7027.3867\ldots$, $S_{50} = 7958.7697\ldots$, $S_{100} = 7999.7875\ldots$, and $S_{200} = 7999.9999\ldots$
11. The partial sums get closer and closer to 8000.
12. Answers will vary.

Exploration 14-3b

1. 20 − 13 = 27 − 20 = · · · = 76 − 69 = 7; 13 + 20 + · · · + 76 = 445
2. 13 + 76 = 20 + 69 = · · · = 41 + 48 = 89; 5 pairs;
   $S_{10} = 5 \cdot 89 = 445$
3. $t_{11} = 76 + 7 = 83$;
   13 + 83 = 20 + 76 = · · · = 41 + 55 = 96
   Now there are $5\frac{1}{2}$ pairs, and $5\frac{1}{2} \cdot 96 = 528 = 13 + 20 + \cdots + 83$.
4. $S_{300} = \dfrac{300}{2}(47 + 3{,}927) = 596{,}100$
5. $t_{47} = 54 + (47 - 1) \cdot 11 = 560$
   $S_{47} = \dfrac{47}{2}(54 + 560) = 14{,}429$
   The program gives the same answer.
6. $\dfrac{100{,}000}{2}(1 + 100{,}000) = 5{,}000{,}050{,}000$
   At the current speed of calculators, the program would take too long.
7. $S_n = \dfrac{n}{2}(t_1 + t_n) = \dfrac{n}{2}[t_1 + [t_1 + d(n-1)]]$
   $= \dfrac{n}{2}[2t_1 + d(n-1)]$
8. $d = 38.2 - 37 = 39.4 - 38.2 = 1.2$;
   $\dfrac{n}{2}[2 \cdot 37 + 1.2(n-1)] = 86{,}697$
   $\Rightarrow \dfrac{n}{2}(74 + 1.2n - 1.2) = 86{,}697$
   $\Rightarrow 1.2n^2 + 72.8n - 173{,}394 = 0$
   $\Rightarrow n = \dfrac{-72.8 \pm \sqrt{72.8^2 - 4(1.2)(-173{,}394)}}{2(1.2)} = 351$
   (because n must be positive); the 351st partial sum
9. Answers will vary.

Exploration 14-3c

1. $\dfrac{21}{7} = \dfrac{63}{21} = \cdots = \dfrac{1701}{567} = 3$; 7 + 21 + · · · + 1701 = 2548
2. $3S_6 = 21 + 63 + 189 + 567 + 1701 + 5103$. They are the same as the terms of the original series, except the first term is missing and there is an added term at the end.
3. They go to 0 by canceling each other out.
4. $S_6 - 3S_6 = t_1 - t_7 = 7 - 7 \cdot 3^{7-1} = 7(1 - 3^6)$
5. $-2S_6 = 7(1 - 3^6) \Rightarrow S_6 = \dfrac{7(1 - 3^6)}{-2} = 2548$
6. $S_n - rS_n = (t_1 + t_1r + t_1r^2 + \cdots + t_1r^{n-1})$
   $- (t_1r + t_1r^2 + \cdots + t_1r^{n-1} + t_1r^n)$
   $\Rightarrow S_n(1 - r) = t_1 - t_1r^n \Rightarrow S_n = \dfrac{t_1(1 - r^n)}{1 - r}$
7. $\dfrac{7(1 - 3^{15})}{1 - 3} = 50{,}221{,}171$. The program gives the same answer.
8. $S_{30} = \dfrac{1{,}000(1 - 1.06^{30})}{1 - 1.06} = 79{,}058.1862\ldots$
   The program gives the same answer.
9. $S_{50} = \dfrac{800(1 - 0.9^{50})}{1 - 0.9} = 7958.7697\ldots$;
   $S_{200} = \dfrac{800(1 - 0.9^{200})}{1 - 0.9} = 7999.9999\ldots$;
   r^n approaches 0.
10. Because r^n approaches 0, S_n approaches
    $\dfrac{800(1 - 0)}{1 - 0.9} = \dfrac{800}{1 - 0.9} = 8000$.
    In general, S_n approaches $\dfrac{t_1}{1 - r}$.
11. Answers will vary.

Exploration 14-3d

1. The coefficients match the values in the seventh row:

   1
   1 1
   1 2 1
   1 3 3 1
   1 4 6 4 1
   1 5 10 10 5 1
   1 6 15 20 15 6 1
   1 7 21 35 35 21 7 1

2. $35 = \dfrac{(\text{coefficient of } t_3)(\text{exponent of } a \text{ in } t_3)}{3} = \dfrac{21 \cdot 5}{3}$
3. The pattern works.
4. $t_3 = \dfrac{\frac{7}{1} \cdot 6}{2}a^5b^2 = \dfrac{7}{1} \cdot \dfrac{6}{2}a^5b^2$
5. $\dfrac{7 \cdot 6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3 \cdot 4} = \dfrac{7 \cdot 6 \cdot 5 \cdot 4 \cdot (3 \cdot 2 \cdot 1)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot (3 \cdot 2 \cdot 1)} = \dfrac{7!}{4!3!}$
6. $\dfrac{7!}{5!2!} = \dfrac{5040}{120 \cdot 2} = 21$