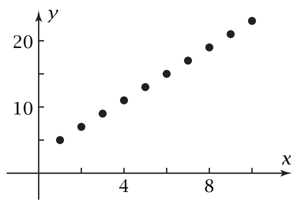
Chapter 14 Sequences and Series

Problem Set 14-1

1. You would have to add nine 2s; $5 + 9 \cdot 2 = 23$; $5 + 99 \cdot 2 = 203$

2.



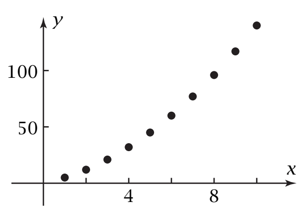
3. Linear

4. $5 + 7 + 9 + \ldots + 21 + 23 = 140$

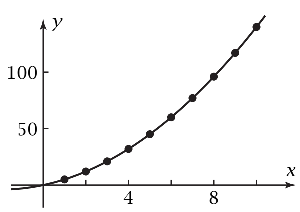
5. $\dfrac{5 + 23}{2} \cdot 10 = 140 = \text{the partial sum}$

   $\dfrac{5 + 203}{2} \cdot 100 = 1040$

6. The partial sums are 5, 12, 21, 32, 45, 60, 77, 96, 117, and 140.



7. Quadratic regression fits exactly:
   $y = x^2 + 4x = 100^2 + 4(100) = 10{,}400$



8. Each term is twice the preceding one. You would have to multiply by 2 nine times, that is, multiply by 2^9.

   $6 \cdot 2^9 = 3072$

9. $6 + 12 + 24 + \ldots + 1536 + 3072 = 6138$

10. Answers will vary.

Problem Set 14-2

Q1. Exponential

Q2. Logarithmic

Q3. Linear

Q4. Direct linear power

Q5. Multiply it by 8.

Q6. A positive or negative whole number, or zero

Q7. -1 or -6

Q8. $5\vec{i} - \vec{j}$

Q9. $5 - i$

Q10. $(5, 210°)$

1. a. Geometric, with common ratio $\frac{36}{27} = \frac{48}{36} = \frac{4}{3}$

   b. $48 \cdot \dfrac{4}{3} = 64$, $64 \cdot \dfrac{4}{3} = 85.\overline{3}$

   c. $t_{100} = 27 \cdot \left(\dfrac{4}{3}\right)^{99} = (6.3139\ldots) \times 10^{13} \approx 63$ trillion

   d. $27 \cdot \left(\dfrac{4}{3}\right)^{n-1} = 849{,}490.02\ldots$

   $\Rightarrow \log 27 + (n - 1)\log\frac{4}{3} = \log 849{,}490.02\ldots$

   $\Rightarrow n = \dfrac{\log(849{,}490.02\ldots) - \log 27}{\log\frac{4}{3}} + 1 = 37$; the 37th term

2. a. Arithmetic, with common difference $31 - 27 = 35 - 31 = 4$

   b. $35 + 4 = 39$, $39 + 4 = 43$

   c. $t_{100} = 27 + 99 \cdot 4 = 423$

   d. $27 + (n - 1) \cdot 4 = 783$

   $\Rightarrow n = \dfrac{783 - 27}{4} + 1 = 190$; the 190th term

3. a. Arithmetic, with common difference $45 - 58 = 32 - 45 = -13$

   b. $32 + (-13) = 19$, $19 + (-13) = 6$

   c. $t = 58 + 99 \cdot (-13) = -1229$

   d. $58 + (n - 1)(-13) = -579 \Rightarrow n = \dfrac{-579 - 58}{-13} + 1 = 50$; the 50th term

4. a. Geometric, with common ratio $\dfrac{90}{100} = \dfrac{81}{90} = \dfrac{9}{10}$

   b. $81 \cdot \dfrac{9}{10} = 72.9$, $72.9 \cdot \dfrac{9}{10} = 65.61$

   c. $t_{100} = 100 \cdot \left(\dfrac{9}{10}\right)^{99} = 0.002951\ldots$

   d. $100 \cdot \left(\dfrac{9}{10}\right)^{n-1} = 3.0903\ldots$

   $\Rightarrow \log 100 + (n - 1)\log\dfrac{9}{10} = \log 3.0903\ldots$

   $\Rightarrow n = \dfrac{\log(3.0903\ldots) - \log 100}{\log\frac{9}{10}} + 1 = 34$; the 34th term

5. a. Geometric, with common ratio $\dfrac{137}{54.8} = \dfrac{342.5}{137} = 2.5$

   b. $(342.5)(2.5) = 856.25$, $(856.25)(2.5) = 2140.625$

   c. $t_{100} = 54.8(2.5)^{99} = 1.3640\ldots \times 10^{41}$

   d. $54.8(2.5)^{n-1} = 3{,}266{,}334.53\ldots$

   $\Rightarrow \log 54.8 + (n - 1)\log 2.5 = \log 3{,}266{,}334.53\ldots$

   $\Rightarrow n = \dfrac{\log(3{,}266{,}334.53\ldots) - \log 54.8}{\log 2.5} = 13$; the 13th term

6. a. Arithmetic, with common difference $79 - 67.3 = 90.7 - 79 = 11.7$

   b. $90.7 + 11.7 = 102.4$, $102.4 + 11.7 = 114.1$

   c. $t_{100} = 67.3 + 99 \cdot (11.7) = 1225.6$

   d. $67.3 + (n - 1)(11.7) = 38{,}490.1$

   $\Rightarrow n = \dfrac{38{,}490.1 - 67.3}{11.7} + 1 = 3285$; the 3285th term

7. a. Geometric, with common ratio $\frac{-45}{50} = \frac{40.5}{-45} = -\frac{9}{10}$

b. $40.5\left(-\frac{9}{10}\right) = -36.45$, $-36.45\left(-\frac{9}{10}\right) = 32.805$

c. $t_{100} = 50\left(-\frac{9}{10}\right)^{99} = -0.001475\ldots$

d. You can't take the logarithm of a negative number, so rewrite the formula as

$(-1)^{n-1} \cdot 50\left(\frac{9}{10}\right)^{n-1}$,

ignore the sign, and solve. Then check whether the answer gives the right sign.

$50\left(\frac{9}{10}\right)^{n-1} = 15.6905\ldots$

$\Rightarrow \log 50 + (n-1)\log\frac{9}{10} = \log 15.6905\ldots$

$\Rightarrow n = \frac{\log(15.9605\ldots) - \log 50}{\log\frac{9}{10}} + 1 = 12$

Since t_{12} would involve the 11th power of a negative number, which would be negative, $n = 12$ is the correct answer.

8. a. Arithmetic, with common difference $-1215.7 - (-1234) = -1197.4 - (-1215.7) = 18.3$

b. $-1197.4 + 18.3 = -1179.1$, $-1179.1 + 18.3 = -1160.8$

c. $t_{100} = -1234 + 99(18.3) = 577.7$

d. $-1234 + (n-1)(18.3) = 2426$

$\Rightarrow \frac{2426 - (-1234)}{18.3} + 1 = 201$; the 201st term

9. a. There is no common ratio. The common differences are 3, 5, 7, 9, 11, 13, 15, 17, 19, . . . , and the common second differences are all 2, so the sequence is quadratic. We can find the formula either by entering 1, 2, 3, . . . into List 1 and the sequence into List 2 and doing quadratic regression, or by solving simultaneously

$$\begin{cases} a(1)^2 + b(1) + c = 0 \\ a(2)^2 + b(2) + c = 3 \\ a(3)^2 + b(3) + c = 8 \end{cases}$$

that is, $\begin{cases} a + b + c = 0 \\ 4a + 2b + c = 3 \\ 9a + 3b + c = 8 \end{cases}$

An even easier way is to hope that the quadratic is factorable and inspect the sequence to see if its terms factor in an obvious pattern.

n	t_n	
1	0	$0 \cdot 2$
2	3	$1 \cdot 3$
3	8	$2 \cdot 4$
4	15	$3 \cdot 5$
5	24	$4 \cdot 6$
n	t_n	$(n-1)(n+1)$

So $t_n = n^2 - 1$.

b. $11^2 - 1 = (11-1)(11+1) = 120$

$12^2 - 1 = (12-1)(12+1) = 143$

c. $t_{100} = 100^2 - 1 = (100-1)(100+1) = 9999$

d. $n^2 - 1 = 3248 \Rightarrow n^2 = 3249 \Rightarrow n = 57$ (because you want the positive value); the 57th term

10. a. There is no common ratio. The differences are 6, 8, 10, 12, 14, 16, . . . , and the second differences are all 2, so the sequence is quadratic. As in Problem 9, see if the terms factor with an obvious pattern.

n	t_n	
1	4	$1 \cdot 4$
2	10	$2 \cdot 5$
3	18	$3 \cdot 6$
4	28	$4 \cdot 7$
5	40	$5 \cdot 8$
n	t_n	$n(n+3)$

So $t_n = n^2 + 3n$.

b. $8^2 + 3 \cdot 8 = 8(8+3) = 88$

$9^2 + 3 \cdot 9 = 9(9+3) = 108$

c. $t_{100} = 100^2 + 3 \cdot 100 = 100(100+3) = 10{,}300$

d. $n^2 + 3n = 178{,}504 \Rightarrow n^2 + 3n - 178{,}504 = 0$

$\Rightarrow (n + 424)(n - 421) = 0 \Rightarrow n = 421$

(because you want the positive value); the 421st term

11. a. Arithmetic, with common difference $(2x - a) - x = (3x - 2a) - (2x - a) = x - a$

b. $(3x - a) + (x - a) = 4x - 3a$; $(4x - 3a) + (x - a) = 5x - 4a$

c. $t_{100} = x + 99(x - a) = 100x - 99a$. Also, it's easy to see more simply that $t_n = nx - (n-1)a$.

d. $x + (n-1)(x - a) = 240 - 239a \Rightarrow n = 240$; the 240th term. This is also easy to see more simply from the fact that $t_n = nx - (n-1)a$.

12. a. Geometric, with common factor $\frac{5\sqrt{2}}{5} = \frac{10}{5\sqrt{2}} = \sqrt{2}$

b. $10 \cdot \sqrt{2} = 10\sqrt{2}$; $(10\sqrt{2}) \cdot \sqrt{2} = 20$

c. $t_{100} = 5(\sqrt{2})^{99} = 3.9086\ldots \times 10^{15}$

d. $5(\sqrt{2})^{n-1} = 20{,}480 \Rightarrow n = 1 + \frac{\log 20{,}480 - \log 5}{\frac{1}{2}\log 2} = 25$; the 25th term

13. Geometric, with common ratio 2

$1 \cdot 2^{n-1} = 1000 \Rightarrow (n-1)\log 2 = \log 1000$

$\Rightarrow n = \frac{\log 1000}{\log 2} + 1 = 10.9657\ldots$;

the 11th square. The king must have started to worry during the second row of squares. $1 \cdot 2^{64-1} = 9.2233\ldots \times 10^{18}$

The king was upset because the number of grains of rice got so large so quickly.

14. $\frac{\$1050.00}{\$1000.00} = \frac{\$1102.50}{\$1050.00} = \frac{\$1157.63}{\$1102.50} = 1.05$, allowing for round-off (this is 5% annual interest). Because n is the number of years after 1799, $t_1 = \$1050$ in 1800; \$1000 in 1799 is actually t_0. So the formula is $t_n = 1050 \cdot (1.05)^{n-1}$, where n is the year minus 1799. You can use the equivalent but simpler $t_n = 1000 \cdot (1.05)^n$.

$1000(1.05)^n = 1{,}000{,}000 \Rightarrow 1.05^n = 1000$

$\Rightarrow n\log 1.05 = 3 \Rightarrow n = \frac{3}{\log 1.05} = 141.5808\ldots$;

142 years later, or 1799 + 142 = 1941. The current value depends on which year the problem is done. Here are the values for several years:

In 2000: $1000(1.05)^{201} = \$18{,}157{,}209.86$

In 2001: $1000(1.05)^{202} = \$19{,}065{,}070.35$

In 2002: $1000(1.05)^{203} = \$20{,}018{,}323.87$

In 2003: $1000(1.05)^{204} = \$21{,}019{,}240.06$

In 2004: $1000(1.05)^{205} = \$22{,}070{,}202.06$

15. a. \$1,267,500, \$1,235,000, \$1,202,500, \$1,170,000, . . .
t_1 = \$1,267,500 after the first year; \$1,300,000 is really t_0. So the formula is $t_n = 1{,}267{,}500 - (n-1)\cdot 32{,}500$. You can use the equivalent but simpler $t_n = 1{,}300{,}000 - n\cdot 32{,}500$, where $n \ge 0$. $1300{,}000 - n\cdot 32{,}500 = 0$
$\Rightarrow n = \dfrac{1{,}300{,}000}{32{,}500} = 40$ yr.
The depreciation function is linear and the scatter plot points lie on a straight line.

b. \$1,170,000, \$1,053,000, \$947,700, \$852,930, . . .
For the same reason as in part a, the formula is $t_n = 1{,}170{,}000\cdot(0.9)^{n-1}$, but you can use $t_n = 1{,}300{,}000\cdot(0.9)^n$ instead.
The business can deduct:
\$130,000 the first year,
\$117,000 the second year,
\$105,300 the third year,
. . .
$\$130{,}000\cdot 0.9^{n-1}$ the nth year.
$\$130{,}000\cdot 0.9^{n-1} < \$32{,}500$
$\Rightarrow n > \dfrac{\log\frac{32{,}500}{130{,}000}}{\log 0.9} = 14.1576\ldots;$
15 yr.

16. a. Arithmetic, with common difference \$2.
$t_{10} = 5 + (10-1)\cdot 2 = \$23$. $5 + (n-1)\cdot 2 = 99$
$\Rightarrow n = \dfrac{99-5}{2} + 1 = 48$; the 48th week

b. $5 + 7 + \ldots + 23 = \$140 = \dfrac{5+23}{2}\cdot 10$

c. $t_{52} = 5 + (52-1)\cdot 2 = \$107$
$5 + 7 + \ldots + 107 = \dfrac{5+107}{2}\cdot 52 = \$2912$

17. 100% − 9% = 91%, 91% of 91% ≈ 82.8%, 91% of 82.8% ≈ 75.4%.
Geometric, with common ratio 0.91. The first washing is t_1 = 91%, so the formula is $t_n = 0.91(0.91)^{n-1}$, but you can use the equivalent and simpler $t_n = 0.91^n$.
$t_{20} = 0.91^{20} = 0.1516\ldots \approx 15.2\%$.
$0.91^n = 0.10 \Rightarrow n\log 0.91 = \log 0.10$
$\Rightarrow n = \dfrac{\log 0.10}{\log 0.91} = 24.4148\ldots;$
25 washings.

18. a. 2, 4, 8. Geometric, with common ratio 2. Since the first generation back is $t_1 = 2$, the formula is $t_n = 2\cdot 2^{n-1}$. You can use the equivalent but simpler $t_n = 2^n$.
$t_{10} = 2^{10} = 1024$
$t_{20} = 2^{20} = 1{,}048{,}576$

b. People must have common ancestors.

19. a. $t_n = t_{n-2} + t_{n-1}$
Each term is the sum of the preceding two.
$t_{11} = 34 + 55 = 89$
$t_{12} = 55 + 89 = 144$
$t_{20} = 6765$

b. 1, 2, 1.5, $1.\overline{6}$, 1.6, 1.625, 1.6153…, 1.6190…, 1.6176…, $1.6\overline{18}$, 1.6179…

c. Answers will vary. The spirals in each direction usually are consecutive Fibonacci numbers.

d. Answers will vary. Leonardo Fibonacci was an Italian mathematician of the late 12th and early 13th centuries. The Fibonacci term t_n is the number of pairs of rabbits there will be in the nth month if you start with one pair and if every pair produces another pair every month but not starting until they are two months old.

20. a. $t_n = n(n-1)(n-2)\cdot \ldots \cdot 3\cdot 2\cdot 1$
The product of all the natural numbers up to and including n. The next two terms are 5,040 and 40,320.

b. $10! = 3{,}628{,}800$, $20! = (2.4329\ldots)\times 10^{18}$
This is quite tedious to calculate by hand. Fortunately, it is built into calculators. The exclamation mark suggests how surprisingly quickly factorials grow.

21. a. To get to step 3, she can take one step from step 2 or two steps from step 1. So, the number of ways to get to step 3 is the number of ways to get to step 1 plus the number of ways to get to step 2. Similarly, the number of ways to get to step 4 is the number of ways to get to step 2 plus the number of ways to get to step 3.

b. $t_n = t_{n-2}$ where $t_2 = 2$ and $t_1 = 1$
$t_{20} = 10{,}946$

c. If you let $t_0 = 1$, then this is the same sequence.

d. $t_{91} = 7.5 \times 10^{18}$

22. a. b_{n-1} is the amount the family owed at the end of the previous month. $0.005b_{n-1}$ is the interest the family pays on the previous month's balance. \$1,074.65 is the amount they paid on the mortgage this month.

b. $b_{12} = 145{,}995$; $\$1{,}074.65\cdot 12 = \$12{,}895.80$;
\$4,005 went toward paying off the mortgage and \$8,890.80 went toward paying interest.

c. The balance will have dropped to zero after the 240th month.

23. $u(40) \approx \$113.56$ and $u(41) \approx \$15.27$, so the balance will be below \$100 in the 41st month, after 40 payments. At this time you will have paid $40\cdot\$100$, or \$4000, so the total is \$4000 + \$15.27 = \$4015.27.

24. a. 56.25, 65.50, 74.75, where common difference is 9.25

b. 6, 12, 24, where common ratio is 2

c. −6, 12, −24, where common ratio is −2

Problem Set 14-3

Q1. 30, 40

Q2. 40, 80

Q3. $\frac{1}{7}, \frac{1}{8}$

Q4. 24, 120

Q5. 320

Q6. 275,612.2467…

Q7. 13

Q8. −5

Q9. Hyperbola

Q10. $\begin{bmatrix} 24 & 13 \\ 26 & 17 \end{bmatrix}$

1. a. $[3 + (1-1)(5)] + [3 + (2-1)(5)] + [3 + (3-1)(5)] + \ldots + [3 + (10-1)(5)] = 3 + 8 + 13 + \ldots + 48$
There is a common difference of 5.

b. $S_{10} = 255 = \dfrac{3+48}{2}\cdot 10$
Yes, the answers are the same.

c. $S_{100} = 25{,}050 = \dfrac{3+498}{2}\cdot 100$

2. a. $5 \cdot 3^{1-1} + 5 \cdot 3^{2-1} + 5 \cdot 3^{3-1} + \ldots + 5 \cdot 3^{6-1}$
$= 5 + 15 + 45 + \ldots + 1215$
The terms have a common ratio of 3.

b. $S_{10} = 1820 = 5 \cdot \dfrac{1 - 3^6}{1 - 3}$
The answers are the same.

c. $S_{20} = 8{,}716{,}961{,}000 = 5 \cdot \dfrac{1 - 3^{20}}{1 - 3}$

3. a. $r = \dfrac{80}{100} = 0.8$; $t_{10} = 100 \cdot 0.8^{10-1} = 13.4217\ldots$ cm;
$S_{10} = 100 \cdot \dfrac{1 - 0.8^{10}}{1 - 0.8} = 446.3129\ldots$ cm

b. The answers are getting closer and closer to 500. You can make the answer as close to 500 as you want by taking the sum of enough terms.

c. $\lim\limits_{n\to\infty} S_n = 100 \cdot \dfrac{1}{1 - 0.8} = 500$ cm

4. a. $\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{6} + \dfrac{1}{8} = 1\dfrac{1}{24}$ cards.
For the next two questions, put the series into a grapher using sequence mode.

b. 31 cards: $\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{6} + \ldots + \dfrac{1}{62} = 2.0136\ldots$

c. $\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{6} + \ldots + \dfrac{1}{104} = 2.2690\ldots$ cards
The series grows very slowly, yet it grows without bounds, as is shown next.

d. $1 > \dfrac{1}{2}, \dfrac{1}{2} = \dfrac{1}{2}, \dfrac{1}{3} + \dfrac{1}{4} > \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{1}{2},$
$\dfrac{1}{5} + \dfrac{1}{6} + \dfrac{1}{7} + \dfrac{1}{8} > \dfrac{1}{8} + \dfrac{1}{8} + \dfrac{1}{8} + \dfrac{1}{8} = \dfrac{1}{2}$, etc.
So the sum is greater than
$\dfrac{1}{2}\left(\dfrac{1}{2} + \dfrac{1}{2} + \dfrac{1}{2} + \ldots\right)$.

5. a. The series is geometric because there is a common ratio, 1.01. The amount at the fifth month is the fifth partial sum because it is the sum of the first five terms of the series.
$100 \cdot \dfrac{1 - 1.01^5}{1 - 1.01} = \$510.10$

b. $100 \cdot \dfrac{1 - 1.01^{12 \cdot 10}}{1 - 1.01} = \$23{,}003.87$;
$\$23{,}003.87 - 120 \cdot \$100 = \$11{,}003.87$

c. $100 \cdot \dfrac{1 - 1.01^n}{1 - 1.01} = 100{,}000$
$\Rightarrow 1 - 1.01^n = \dfrac{(1 - 1.01)100{,}000}{100} = -10$
$\Rightarrow 1.01^n = 11 \Rightarrow n = \dfrac{\log 11}{\log 1.01} = 240.9864\ldots$
241 months

6. a. $x\left(1 + \dfrac{0.06}{12}\right)^{12 \cdot 10} = 10{,}000 \Rightarrow x = \dfrac{10{,}000}{1.005^{120}} = \$5{,}496.33$

b. $x \cdot \dfrac{1 - 1.005^{12 \cdot 10}}{1 - 1.005} = 10{,}000$
$\Rightarrow x = 10{,}000 \cdot \dfrac{1 - 1.005}{1 - 1.005^{120}} = 61.0205\ldots$;
\$61.02 per month yields
$(61.02)\dfrac{1 - 1.005^{120}}{1 - 1.005} = \$9{,}999.92$;
\$61.03 per month yields
$(61.03)\dfrac{1 - 1.005^{120}}{1 - 1.005} = \$10{,}001.56$

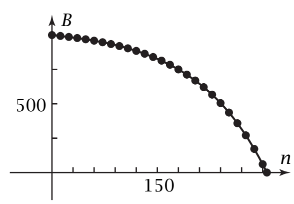
7. a.

n	P	I_n	P_n	B_n
4	1,050.00	998.48	51.52	99,796.97

b. $B_1 = B_0(1 + i) - P$
$B_2 = B_1(1 + i) - P = (B_0(1 + i) - P)(1 + i) - P$
$= B_0(1 + i)^2 - P(1 + i) - P$
$B_3 = B_2(1 + i) - P$
$= (B_0(1 + i)^2 - P(1 + i) - P)(1 + i) - P$
$= B_0(1 + i)^3 - P(1 + i)^2 - P(1 + i) - P$
$B_4 = B_3(1 + i) - P$
$= B_0(1 + i)^4 - P(1 + i)^3 - P(1 + i)^2$
$- P(1 + i) - P$
$B_4 = B_0(1 + i)^4 - P(1 + i)^3 - P(1 + i)^2$
$- P(1 + i) - P$
$= B_0(1 + i)^4$
$- P(1 + (1 + i) + (1 + i)^2 + (1 + i)^3)$
$= B_0(1 + i)^4 - P\dfrac{1 - (1 + i)^4}{1 - (1 + i)}$
$= B_0(1 + i)^4 - P\dfrac{1 - (1 + i)^4}{-i}$
$= B_0(1 + i)^4 + \dfrac{P}{i}\left(1 - (1 + i)^4\right)$

c. $B_n = B_0(1 + i)^n + \dfrac{P}{i}\left(1 - (1 + i)^n\right)$
$B_n = 0 \Rightarrow B_0(1 + i)^n = -\dfrac{P}{i}\left(1 - (1 + i)^n\right)$
$\dfrac{-iB_0}{P} = \dfrac{1 - (1 + i)^n}{(1 + i)^n} = (1 + i)^{-n} - 1$
$n = -\dfrac{\log\left(1 - \frac{iB_0}{P}\right)}{\log(1 + i)} = 305.9719\ldots$
306 months

d.



False: Halfway through the mortgage (month 153), the balance is still \$82,083.93, about 82% of the original amount.

8. a. By long division,

$$\begin{array}{r l} & 1 + r + r^2 + r^3 + \ldots \\ 1 - r \,\big)\!\! & 1 \\ & \underline{1 - r} \\ & \quad r \\ & \quad \underline{r - r^2} \\ & \qquad r^2 \\ & \qquad \underline{r^2 - r^3} \\ & \qquad\quad r^3 \\ & \qquad\quad \ldots \end{array}$$

b. Answers will vary. Inverse operations are extremely common in mathematics. One example is doing subtraction (or division) by "guessing" addition (or multiplication).

9. a. $(0.4 + 0.6)^5$
$= {}_5C_5(0.4)^5(0.6)^0 + {}_5C_4(0.4)^4(0.6)^1$
$+ {}_5C_3(0.4)^3(0.6)^2 + {}_5C_2(0.4)^2(0.6)^3$
$+ {}_5C_1(0.4)^1(0.6)^4 + {}_5C_0(0.4)^0(0.6)^5$
$= 0.01024 + 0.07680$
$+ 0.23040 + 0.34560$
$+ 0.25920 + 0.07776$

b. ${}_5C_2(0.4)^2(0.6)^3 = 0.34560$, about 34.6%

c. Letting X = the number of "point-ups" out of five flips, the first four terms of the series,

$P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$
$= 0.01024 + 0.07680 + 0.23040 + 0.3456$
$= 0.66304$, about 66.3%

d. Letting X = the number of "point-ups" out of ten flips,

$P(X \le 6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)$
$= 1 - [P(X=7) + P(X=8) + P(X=9) + P(X=10)]$
$= 1 - {}_{10}C_3(0.4)^3(0.6)^7 - {}_{10}C_2(0.4)^2(0.6)^8 - {}_{10}C_1(0.4)^1(0.6)^9 - {}_{10}C_0(0.4)^0(0.6)^{10}$
$= 0.6177\ldots$, about 61.8%, not the same probability

10. a. $4 \cdot 3 = 12$ sides; $12 \cdot 1 = 12$ units

$4 \cdot 12 = 48$ sides; $48 \cdot \frac{1}{3} = 16$ units

Geometric, with common ratio $\frac{16}{12} = \frac{4}{3}$

$r > 1$, so perimeter $\to 2$

b. First iteration: $b = 1$, $h = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$,

$A_\Delta = \frac{1}{2}bh = \frac{\sqrt{3}}{4}$ units^2.

Total shaded area $= 3 \cdot \frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{4}$ units^2.

Second iteration: The added triangles are $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ as large, so

$A_\Delta = \frac{1}{9} \cdot \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{36}$ units^2.

Total added area $= 12 \cdot \frac{\sqrt{3}}{36} = \frac{\sqrt{3}}{3}$ units^2.

This is a geometric series with common ratio $\frac{\sqrt{3}}{3} \div \frac{3\sqrt{3}}{4} = \frac{4}{9}$, so $\lim_{n\to\infty} S_n = \frac{3\sqrt{3}}{4} \cdot \frac{1}{1 - \frac{4}{9}} = \frac{27\sqrt{3}}{20}$ units^2
$= 2.3382\ldots$ units^2.

11. $[2(1) + 7] + [2(2) + 7] + [2(3) + 7] + [2(4) + 7] + [2(5) + 7]$
$= 9 + 11 + 13 + 15 + 17 = 65$

12. $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2$
$= 1 + 4 + 9 + 16 + 25 + 36 + 49 = 140$

13. $3^1 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6$
$= 3 + 9 + 27 + 81 + 243 + 729 = 1092$

14. $1! + 2! + 3! + 4! + 5! + 6! = 1 + 2 + 6 + 24 + 120 + 720 = 873$

15. Geometric, $r = \frac{10}{2} = \frac{50}{10} = 5$;

$S_{11} = 2 \cdot \frac{1 - 5^{11}}{1 - 5} = 24{,}414{,}062$

16. Arithmetic,
$d = 131 - 97 = 165 - 131 = 34$;
$t_{37} = 97 + (37 - 1) \cdot 34 = 1321$;
$S_{37} = \frac{37}{2}(97 + 1321) = 26{,}233$

17. Arithmetic, $d = 31.6 - 24 = 39.2 - 31.6 = 7.6$;
$t_{54} = 24 + (54 - 1)(7.6) = 426.8$;
$S_{54} = \frac{54}{2}(24 + 426.8) = 12{,}171.6$

18. Geometric, $r = \frac{54}{36} = \frac{81}{54} = \frac{3}{2}$;

$S_{29} = 36 \cdot \frac{1 - \left(\frac{3}{2}\right)^{2.9}}{1 - \frac{3}{2}} = 9{,}203{,}978.8431\ldots$

19. Arithmetic, $d = 960 - 1000 = 920 - 960 = -40$;
$t_{78} = 1000 + (78 - 1)(-40) = -2080$;
$S_{78} = \frac{78}{2}[1000 + (-2080)] = -42{,}120$

20. Geometric, $r = \frac{900}{1000} = \frac{810}{900} = 0.9$;

$S_{22} = 1000 \cdot \frac{1 - 0.9^{22}}{1 - 0.9} = 9015.2290\ldots$

21. Geometric, $r = \frac{-150}{50} = \frac{450}{-150} = -3$;

$S_{10} = 50 \cdot \frac{1 - (-3)^{10}}{1 - (-3)} = 738{,}100$

22. Geometric, $r = \frac{-52}{32.5} = \frac{83.2}{-52} = -1.6$;

$S_{41} = (32.5) \cdot \frac{1 - (-1.6)^{41}}{1 - (-1.6)} = 2{,}923{,}003{,}287.1622\ldots$

23. Arithmetic, $d = 43 - 32 = 54 - 43 = 11$;

$S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}\left[t_1 + [t_1 + (n-1)d]\right]$
$= \frac{n}{2}[2t_1 + (n-1)d] = \frac{n}{2}[2 \cdot 32 + (n-1) \cdot 11]$

$\frac{n}{2}(11n + 53) = 4407 \Rightarrow 11n^2 + 53n - 8814 = 0$

$\Rightarrow n = \frac{-53 \pm \sqrt{53^2 - 4(11)(-8814)}}{2(11)} = 26$

(because n must be a positive integer)

24. Geometric, $r = \frac{26}{13} = \frac{52}{26} = 2$; $S_n = 13 \cdot \frac{1 - 2^n}{1 - 2}$
$= 13(2^n - 1) = 425{,}971 \Rightarrow 2^n = 32{,}768$
$\Rightarrow n = \frac{\log 32{,}768}{\log 2} = 15$

25. Geometric, $r = \frac{30}{18} = \frac{50}{30} = \frac{5}{3}$; $S_n = 18 \cdot \frac{1 - \left(\frac{5}{3}\right)^n}{1 - \frac{5}{3}}$

$= 27\left[\left(\frac{5}{3}\right)^n - 1\right] \approx 443{,}088 \Rightarrow \left(\frac{5}{3}\right)^n \approx 16{,}411.\overline{6}$

$\Rightarrow n \approx \frac{\log 16{,}411.\overline{6}}{\log \frac{5}{3}} = 19.0001192\ldots$; $n = 19$

26. Arithmetic, $d = 101 - 97 = 105 - 101 = 4$;

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$ (see Problem 23)

$= \frac{n}{2}[2 \cdot 97 + (n-1) \cdot 4] = 2n^2 + 95n = 21{,}663$

$\Rightarrow n = \frac{-95 \pm \sqrt{95^2 - 4(2)(-21{,}663)}}{2(2)} = 83$

(because n must be a positive integer)

27. Arithmetic, $d = 91 - 97 = 85 - 91 = -6$;

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$ (see Problem 23)

$= \frac{n}{2}[2 \cdot 97 + (n-1)(-6)] = 100n - 3n^2 = 217$

$\Rightarrow n = \frac{-(-100) \pm \sqrt{(-100)^2 - 4(3)(217)}}{2(3)} = 31$

(because n must be a positive integer)