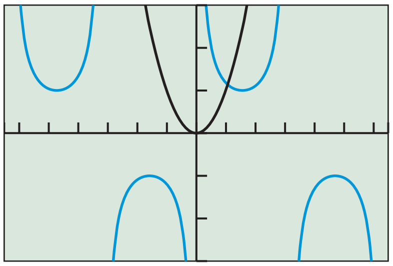
EXAMPLE 4 Solving a Trigonometric Equation Graphically

Find the smallest positive number x such that $x^2 = \csc x$.

SOLUTION There is no algebraic attack that looks hopeful, so we solve this equation graphically. The intersection point of the graphs of $y = x^2$ and $y = \csc x$ that has the smallest positive x-coordinate is shown in Figure 4.55. We use the grapher to determine that $x \approx 1.068$.



[–6.5, 6.5] by [–3, 3]

FIGURE 4.55 A graphical solution of a trigonometric equation. (Example 3)

Now try Exercise 39.

ARE COSECANT CURVES PARABOLAS?

Figure 4.55 shows a parabola intersecting one of the infinite number of U-shaped curves that make up the graph of the cosecant function. In fact, the parabola intersects *all* of those curves that lie above the x-axis, since the parabola must spread out to cover the entire domain of $y = x^2$, which is all real numbers! The cosecant curves do not keep spreading out, as they are hemmed in by asymptotes. That means that the U-shaped curves in the cosecant function are not parabolas.

ASSIGNMENT GUIDE

Ex. 3–48, multiples of 3, 51–56 all

COOPERATIVE LEARNING

Group Activity: Ex. 63

To close this section, we summarize the properties of the six basic trigonometric functions in tabular form. The "n" that appears in several places should be understood as taking on all possible integer values: $0, \pm1, \pm2, \pm3, \ldots$.

Summary: Basic Trigonometric Functions

Function	Period	Domain	Range	Asymptotes	Zeros	Even/Odd
$\sin x$	2π	All reals	$[-1, 1]$	None	$n\pi$	Odd
$\cos x$	2π	All reals	$[-1, 1]$	None	$\pi/2 + n\pi$	Even
$\tan x$	π	$x \neq \pi/2 + n\pi$	All reals	$x = \pi/2 + n\pi$	$n\pi$	Odd
$\cot x$	π	$x \neq n\pi$	All reals	$x = n\pi$	$\pi/2 + n\pi$	Odd
$\sec x$	2π	$x \neq \pi/2 + n\pi$	$(-\infty, -1] \cup [1, \infty)$	$x = \pi/2 + n\pi$	None	Even
$\csc x$	2π	$x \neq n\pi$	$(-\infty, -1] \cup [1, \infty)$	$x = n\pi$	None	Odd

QUICK REVIEW 4.5 *(For help, go to Sections 1.2, 2.6, and 4.3.)*

In Exercises 1–4, state the period of the function.

1. $y = \cos 2x$ π **2.** $y = \sin 3x$ $2\pi/3$

3. $y = \sin \frac{1}{3}x$ 6π **4.** $y = \cos \frac{1}{2}x$ 4π

In Exercises 5–8, find the zeros and vertical asymptotes of the function.

5. $y = \dfrac{x-3}{x+4}$ **6.** $y = \dfrac{x+5}{x-1}$

7. $y = \dfrac{x+1}{(x-2)(x+2)}$ **8.** $y = \dfrac{x+2}{x(x-3)}$

In Exercises 9 and 10, tell whether the function is odd, even, or neither.

9. $y = x^2 + 4$ even **10.** $y = \dfrac{1}{x}$ odd

5. Zero: 3; asymptote: $x = -4$ **6.** Zero: -5; asymptote: $x = 1$
7. Zero: -1; asymptotes: $x = 2$ and $x = -2$
8. Zero: -2; asymptotes: $x = 0$ and $x = 3$