

**Accelerated Math 3**  
**Using Trigonometric Identities**

Use the double-angle and half-angle formulas for sine and cosine to answer the following questions.

**Challenge Problems**

1. Show that  $\frac{\cos(2\theta)}{2} = \cos\theta$  is false whenever  $0 < \theta < \frac{\pi}{2}$
2. Show that  $\frac{\cos(2\theta)}{2} = \cos\theta$  is true for one value of  $\theta$  in the interval  $\frac{\pi}{2} < \theta < \pi$
3. Explain why  $|\cos\theta \sin\theta| \leq \frac{1}{2}$  for every angle  $\theta$
4. Without doing any algebraic manipulations, explain why  $(2\cos^2\theta - 1)^2 + (2\cos\theta \sin\theta)^2 = 1$ .
5. Find angles  $u$  and  $v$  such that  $\cos(2u) = \cos(2v)$  but  $\cos(u) \neq \cos(v)$ .

Use the addition and subtraction formulas for sine and cosine to answer the following problems (give exact expressions for all of the following unless otherwise indicated).

**Straightforward Problems**

Given the following exact expressions for the following trig functions, find the requested quantities.

$$\begin{aligned} \cos 22.5^\circ &= \frac{\sqrt{2+\sqrt{2}}}{2} & \text{and} & & \sin 22.5^\circ &= \frac{\sqrt{2-\sqrt{2}}}{2} \\ \cos 18^\circ &= \frac{\sqrt{\sqrt{5}+5}}{8} & \text{and} & & \sin 18^\circ &= \frac{\sqrt{5}-1}{4} \end{aligned}$$

6.  $\cos(82.5^\circ)$
7.  $\cos(48^\circ)$
8.  $\sin(82.5^\circ)$
9.  $\sin(48^\circ)$
10.  $\cos(37.5^\circ)$
11.  $\cos(12^\circ)$
12. Evaluate  $\cos\left(\frac{\pi}{6} + \cos^{-1}\left(\frac{3}{4}\right)\right)$
13. Evaluate  $\sin\left(\cos^{-1}\left(\frac{1}{4}\right) + \tan^{-1}(2)\right)$
14. Find a formula for  $\cos\left(\theta + \frac{\pi}{2}\right)$
15. Find a formula for  $\sin\left(\theta - \frac{\pi}{4}\right)$
16. Find a formula for  $\tan\left(\theta - \frac{\pi}{2}\right)$

**Challenge Problems**

17. Show that  $\frac{\sqrt{6} + \sqrt{2}}{4} = \frac{\sqrt{2 + \sqrt{3}}}{2}$  without using a calculator and without showing that both expressions are equal to  $\cos(15^\circ)$
18. Show that  $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$  Hint:  $\cos(3\theta) = \cos(2\theta + \theta)$
19. Show that  $\cos u \cos v = \frac{1}{2}(\cos(u+v) + \cos(u-v))$
20. (a) If  $u = \tan^{-1} 2$  and  $v = \tan^{-1} 3$ , show that  $\tan(u+v) = -1$   
(b) Assume the same values for  $u$  and  $v$  from (a). Use part (a) to explain why  $u+v = \frac{3\pi}{4}$   
(c) Using part (b), derive the beautiful equation  $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$