SECTION 6.5 EXERCISES

In Exercises 1 and 2, **(a)** complete the table for the polar equation, and **(b)** plot the corresponding points.

1. $r = 3 \cos 2\theta$

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$
r	3	0	-3	0	3	0	-3	0

2. $r = 2 \sin 3\theta$

θ	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
r	0	2	0	-2	0	2	0

In Exercises 3–6, draw a graph of the rose curve. State the smallest θ-interval $(0 \le \theta \le k)$ that will produce a complete graph.

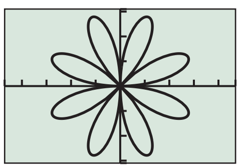
3. $r = 3 \sin 3\theta$

4. $r = -3 \cos 2\theta$

5. $r = 3 \cos 2\theta$

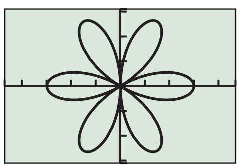
6. $r = 3 \sin 5\theta$

Exercises 7 and 8 refer to the curves in the given figure.



[–4.7, 4.7] by [–3.1, 3.1]

(a)



[–4.7, 4.7] by [–3.1, 3.1]

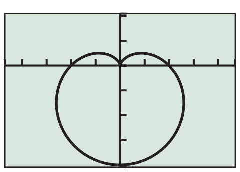
(b)

7. The graphs of which equations are shown? r_3 is graph (b).

$r_1 = 3 \cos 6\theta \quad r_2 = 3 \sin 8\theta \quad r_3 = 3|\cos 3\theta|$

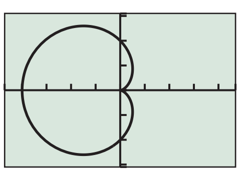
8. Use trigonometric identities to explain which of these curves is the graph of $r = 6 \cos 2\theta \sin 2\theta$. (a)

In Exercises 9–12, match the equation with its graph without using your graphing calculator.



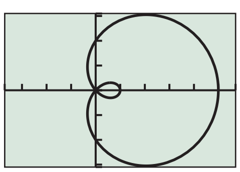
[–4.7, 4.7] by [–4.1, 2.1]

(a)



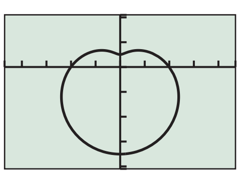
[–4.7, 4.7] by [–3.1, 3.1]

(b)



[–3.7, 5.7] by [–3.1, 3.1]

(c)



[–4.7, 4.7] by [–4.1, 2.1]

(d)

9. Does the graph of $r = 2 + 2 \sin \theta$ or $r = 2 - 2 \cos \theta$ appear in the figure? Explain. Graph (b) is $r = 2 - 2 \cos \theta$.

10. Does the graph of $r = 2 + 3 \cos \theta$ or $r = 2 - 3 \cos \theta$ appear in the figure? Explain. Graph (c) is $r = 2 + 3 \cos \theta$.

11. Is the graph in (a) the graph of $r = 2 - 2 \sin \theta$ or $r = 2 + 2 \cos \theta$? Explain. Graph (a) is $r = 2 - 2 \sin \theta$.

12. Is the graph in (d) the graph of $r = 2 + 1.5 \cos \theta$ or $r = 2 - 1.5 \sin \theta$? Explain. Graph (d) is $r = 2 - 1.5 \sin \theta$.

In Exercises 13–20, use the polar symmetry tests to determine if the graph is symmetric about the x-axis, the y-axis, or the origin.

13. $r = 3 + 3 \sin \theta$

14. $r = 1 + 2 \cos \theta$

15. $r = 4 - 3 \cos \theta$

16. $r = 1 - 3 \sin \theta$

17. $r = 5 \cos 2\theta$

18. $r = 7 \sin 3\theta$

19. $r = \dfrac{3}{1 + \sin \theta}$

20. $r = \dfrac{2}{1 - \cos \theta}$

In Exercises 21–24, identify the points for $0 \le \theta \le 2\pi$ where maximum r-values occur on the graph of the polar equation.

21. $r = 2 + 3 \cos \theta$

22. $r = -3 + 2 \sin \theta$

23. $r = 3 \cos 3\theta$

24. $r = 4 \sin 2\theta$

In Exercises 25–44, analyze the graph of the polar curve.

25. $r = 3$

26. $r = -2$

27. $\theta = \pi/3$

28. $\theta = -\pi/4$

29. $r = 2 \sin 3\theta$

30. $r = -3 \cos 4\theta$

31. $r = 5 + 4 \sin \theta$

32. $r = 6 - 5 \cos \theta$

33. $r = 4 + 4 \cos \theta$

34. $r = 5 - 5 \sin \theta$

35. $r = 5 + 2 \cos \theta$

36. $r = 3 - \sin \theta$

37. $r = 2 + 5 \cos \theta$

38. $r = 3 - 4 \sin \theta$

39. $r = 1 - \cos \theta$

40. $r = 2 + \sin \theta$

41. $r = 2\theta$

42. $r = \theta/4$

43. $r^2 = \sin 2\theta,\ 0 \le \theta \le 2\pi$

44. $r^2 = 9 \cos 2\theta,\ 0 \le \theta \le 2\pi$

In Exercises 45–48, find the length of each petal of the polar curve.

45. $r = 2 + 4 \sin 2\theta$

46. $r = 3 - 5 \cos 2\theta$

47. $r = 1 - 4 \cos 5\theta$

48. $r = 3 + 4 \sin 5\theta$

In Exercises 49–52, select the two equations whose graphs are the same curve. Then, even though the graphs of the equations are identical, describe how the two paths are different as θ increases from 0 to 2π.

49. $r_1 = 1 + 3 \sin \theta, \quad r_2 = -1 + 3 \sin \theta, \quad r_3 = 1 - 3 \sin \theta$

50. $r_1 = 1 + 2 \cos \theta, \quad r_2 = -1 - 2 \cos \theta, \quad r_3 = -1 + 2 \cos \theta$

51. $r_1 = 1 + 2 \cos \theta, \quad r_2 = 1 - 2 \cos \theta, \quad r_3 = -1 - 2 \cos \theta$

52. $r_1 = 2 + 2 \sin \theta, \quad r_2 = -2 + 2 \sin \theta, \quad r_3 = 2 - 2 \sin \theta$

In Exercises 53–56, **(a)** describe the graph of the polar equation, **(b)** state any symmetry that the graph possesses, and **(c)** state its maximum r-value if it exists.

53. $r = 2\sin^2 2\theta + \sin 2\theta$

54. $r = 3\cos 2\theta - \sin 3\theta$

55. $r = 1 - 3\cos 3\theta$

56. $r = 1 + 3\sin 3\theta$

57. Group Activity Analyze the graphs of the polar equations $r = a\cos n\theta$ and $r = a\sin n\theta$ when n is an even integer.

58. Revisiting Example 4 Use the polar symmetry tests to prove that the graph of the curve $r = 3\sin 4\theta$ is symmetric about the y-axis and the origin.

59. Writing to Learn Revisiting Example 5 Confirm the range stated for the polar function $r = 3 - 3\sin\theta$ of Example 5 by graphing $y = 3 - 3\sin x$ for $0 \le x \le 2\pi$. Explain why this works.

60. Writing to Learn Revisiting Example 6 Confirm the range stated for the polar function $r = 2 + 3\cos\theta$ of Example 6 by graphing $y = 2 + 3\cos x$ for $0 \le x \le 2\pi$. Explain why this works.

Standardized Test Questions

61. True or False A polar curve is always bounded. Justify your answer. False. The spiral $r = \theta$ is unbounded.

62. True or False The graph of $r = 2 + \cos\theta$ is symmetric about the x-axis. Justify your answer.

In Exercises 63–66, solve the problem without using a calculator.

63. Multiple Choice Which of the following gives the number of petals of the rose curve $r = 3\cos 2\theta$? D

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 6

64. Multiple Choice Which of the following describes the symmetry of the rose graph of $r = 3\cos 2\theta$? D

(A) only the x-axis

(B) only the y-axis

(C) only the origin

(D) the x-axis, the y-axis, the origin

(E) Not symmetric about the x-axis, the y-axis, or the origin

65. Multiple Choice Which of the following is a maximum r-value for $r = 2 - 3\cos\theta$? B

(A) 6 **(B)** 5 **(C)** 3 **(D)** 2 **(E)** 1

66. Multiple Choice Which of the following is the number of petals of the rose curve $r = 5\sin 3\theta$? B

(A) 1 **(B)** 3 **(C)** 6 **(D)** 10 **(E)** 15

Explorations

67. Analyzing Rose Curves Consider the polar equation $r = a\cos n\theta$ for n, an odd integer.

(a) Prove that the graph is symmetric about the x-axis.

(b) Prove that the graph is not symmetric about the y-axis.

(c) Prove that the graph is not symmetric about the origin.

(d) Prove that the maximum r-value is $|a|$.

(e) Analyze the graph of this curve.

68. Analyzing Rose Curves Consider the polar equation $r = a\sin n\theta$ for n an odd integer.

(a) Prove that the graph is symmetric about the y-axis.

(b) Prove that the graph is not symmetric about the x-axis.

(c) Prove that the graph is not symmetric about the origin.

(d) Prove that the maximum r-value is $|a|$.

(e) Analyze the graph of this curve.

69. Extended Rose Curves The graphs of $r_1 = 3\sin((5/2)\theta)$ and $r_2 = 3\sin((7/2)\theta)$ may be called rose curves.

(a) Determine the smallest θ-interval that will produce a complete graph of r_1; of r_2.

(b) How many petals does each graph have?

Extending the Ideas

In Exercises 70–72, graph each polar equation. Describe how they are related to each other.

70. (a) $r_1 = 3\sin 3\theta$ **(b)** $r_2 = 3\sin 3\left(\theta + \frac{\pi}{12}\right)$

(c) $r_3 = 3\sin 3\left(\theta + \frac{\pi}{4}\right)$

71. (a) $r_1 = 2\sec\theta$ **(b)** $r_2 = 2\sec\left(\theta - \frac{\pi}{4}\right)$

(c) $r_3 = 2\sec\left(\theta - \frac{\pi}{3}\right)$

72. (a) $r_1 = 2 - 2\cos\theta$ **(b)** $r_2 = r_1\left(\theta + \frac{\pi}{4}\right)$

(c) $r_3 = r_1\left(\theta + \frac{\pi}{3}\right)$

73. Writing to Learn Describe how the graphs of $r = f(\theta)$, $r = f(\theta + \alpha)$, and $r = f(\theta - \alpha)$ are related. Explain why you think this generalization is true.