QUICK REVIEW 6.4 *(For help, go to Sections P.2, 4.3, and 5.6.)*

In Exercises 1 and 2, determine the quadrants containing the terminal side of the angles.

1. **(a)** $5\pi/6$ II **(b)** $-3\pi/4$ III

2. **(a)** $-300°$ I **(b)** $210°$ III

In Exercises 3–6, find a positive and a negative angle coterminal with the given angle.

3. $-\pi/4$ $7\pi/4, -9\pi/4$

4. $\pi/3$ $7\pi/3, -5\pi/3$

5. $160°$ $520°, -200°$

6. $-120°$ $240°, -480°$

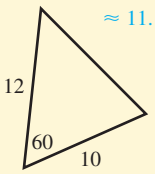
In Exercises 7 and 8, write a standard form equation for the circle.

7. Center (3, 0) and radius 2

8. Center (0, −4) and radius 3

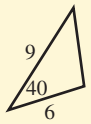
In Exercises 9 and 10, use The Law of Cosines to find the measure of the third side of the given triangle.

9.



≈ 11.14

10.



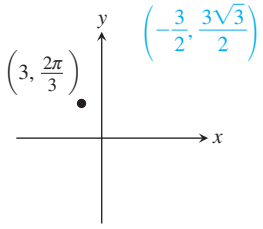
≈ 5.85

7. $(x-3)^2 + y^2 = 4$ **8.** $x^2 + (y+4)^2 = 9$

SECTION 6.4 EXERCISES

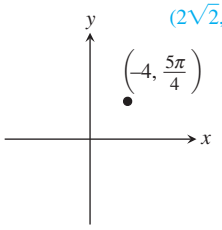
In Exercises 1–4, the polar coordinates of a point are given. Find its rectangular coordinates.

1.



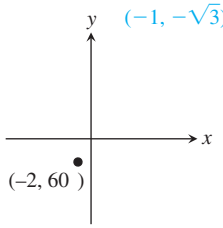
$\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

2.



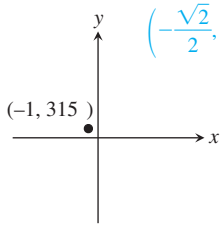
$(2\sqrt{2}, 2\sqrt{2})$

3.



$(-1, -\sqrt{3})$

4.



$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

In Exercises 5 and 6, **(a)** complete the table for the polar equation and **(b)** plot the corresponding points.

5. $r = 3 \sin \theta$

θ	$\pi/4$	$\pi/2$	$5\pi/6$	π	$4\pi/3$	2π
r	$3\sqrt{2}/2$	3	3/2	0	$-3\sqrt{3}/2$	0

6. $r = 2 \csc \theta$

θ	$\pi/4$	$\pi/2$	$5\pi/6$	π	$4\pi/3$	2π
r	$2\sqrt{2}$	2	4	und.	$-4\sqrt{3}/3$	und.

In Exercises 7–14, plot the point with the given polar coordinates.

7. $(3, 4\pi/3)$ **8.** $(2, 5\pi/6)$ **9.** $(-1, 2\pi/5)$

10. $(-3, 17\pi/10)$ **11.** $(2, 30°)$ **12.** $(3, 210°)$

13. $(-2, 120°)$ **14.** $(-3, 135°)$

In Exercises 15–22, find the rectangular coordinates of the point with given polar coordinates.

15. $(1.5, 7\pi/3)$ **16.** $(2.5, 17\pi/4)$

17. $(-3, -29\pi/7)$ **18.** $(-2, -14\pi/5)$ (1.62, 1.18)

19. $(-2, \pi)$ (2, 0) **20.** $(1, \pi/2)$ (0, 1)

21. $(2, 270°)$ (0, −2) **22.** $(-3, 360°)$ (−3, 0)

In Exercises 23–26, polar coordinates of point P are given. Find all of its polar coordinates.

23. $P = (2, \pi/6)$ **24.** $P = (1, -\pi/4)$

25. $P = (1.5, -20°)$ **26.** $P = (-2.5, 50°)$

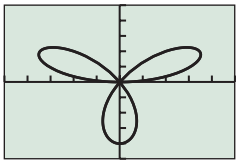
In Exercises 27–30, rectangular coordinates of point P are given. Find all polar coordinates of P that satisfy

(a) $0 \le \theta \le 2\pi$ **(b)** $-\pi \le \theta \le \pi$ **(c)** $0 \le \theta \le 4\pi$

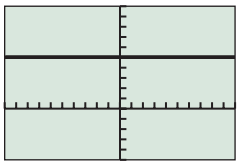
27. $P = (1, 1)$ **28.** $P = (1, 3)$

29. $P = (-2, 5)$ **30.** $P = (-1, -2)$

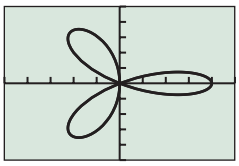
In Exercises 31–34, use your grapher to match the polar equation with its graph.



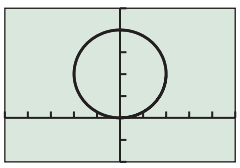
(a)



(b)



(c)



(d)

31. $r = 5 \csc \theta$ (b)

32. $r = 4 \sin \theta$ (d)

33. $r = 4 \cos 3\theta$ (c)

34. $r = 4 \sin 3\theta$ (a)

In Exercises 35–42, convert the polar equation to rectangular form and identify the graph. Support your answer by graphing the polar equation.

35. $r = 3 \sec \theta$

36. $r = -2 \csc \theta$

37. $r = -3 \sin \theta$

38. $r = -4 \cos \theta$

39. $r \csc \theta = 1$

40. $r \sec \theta = 3$

41. $r = 2 \sin \theta - 4 \cos \theta$

42. $r = 4 \cos \theta - 4 \sin \theta$

In Exercises 43–50, convert the rectangular equation to polar form. Graph the polar equation.

43. $x = 2$

44. $x = 5$

45. $2x - 3y = 5$

46. $3x + 4y = 2$

47. $(x - 3)^2 + y^2 = 9$

48. $x^2 + (y - 1)^2 = 1$

49. $(x + 3)^2 + (y + 3)^2 = 18$

50. $(x - 1)^2 + (y + 4)^2 = 17$

51. Tracking Airplanes The location, given in polar coordinates, of two planes approaching the Vicksburg airport are (4 mi, 12°) and (2 mi, 72°). Find the distance between the airplanes. $2\sqrt{3} \approx 3.46$ mi

52. Tracking Ships The location of two ships from Mays Landing Lighthouse, given in polar coordinates, are (3 mi, 170°) and (5 mi, 150°). Find the distance between the ships. ≈ 2.41 mi

53. Using Polar Coordinates in Geometry A square with sides of length a and center at the origin has two sides parallel to the x-axis. Find polar coordinates of the vertices.

54. Using Polar Coordinates in Geometry A regular pentagon whose center is at the origin has one vertex on the positive x-axis at a distance a from the center. Find polar coordinates of the vertices.

Standardized Test Questions

55. True or False Every point in the plane has exactly two polar coordinates. Justify your answer.

56. True or False If r_1 and r_2 are not 0, and if (r_1, θ) and $(r_2, \theta + \pi)$ represent the same point in the plane, then $r_1 = -r_2$. Justify your answer.

In Exercises 57–60, solve the problem without using a calculator.

57. Multiple Choice If $r \neq 0$, which of the following polar coordinate pairs represents the same point as the point with polar coordinates (r, θ)? C

(A) $(-r, \theta)$ **(B)** $(-r, \theta + 2\pi)$ **(C)** $(-r, \theta + 3\pi)$
(D) $(r, \theta + \pi)$ **(E)** $(r, \theta + 3\pi)$

58. Multiple Choice Which of the following are the rectangular coordinates of the point with polar coordinate $(-2, -\pi/3)$? C

(A) $(-\sqrt{3}, 1)$ **(B)** $(-1, -\sqrt{3})$ **(C)** $(-1, \sqrt{3})$
(D) $(1, -\sqrt{3})$ **(E)** $(1, \sqrt{3})$

59. Multiple Choice Which of the following polar coordinate pairs represent the same point as the point with polar coordinates $(2, 110°)$? A

(A) $(-2, -70°)$ **(B)** $(-2, 110°)$ **(C)** $(-2, -250°)$
(D) $(2, -70°)$ **(E)** $(2, 290°)$

60. Multiple Choice Which of the following polar coordinate pairs does *not* represent the point with rectangular coordinates $(-2, -2)$? E

(A) $(2\sqrt{2}, -135°)$ **(B)** $(2\sqrt{2}, 225°)$ **(C)** $(-2\sqrt{2}, -315°)$
(D) $(-2\sqrt{2}, 45°)$ **(E)** $(-2\sqrt{2}, 135°)$

Explorations

61. Polar Distance Formula Let P_1 and P_2 have polar coordinates (r_1, θ_1) and (r_2, θ_2), respectively.

(a) If $\theta_1 - \theta_2$ is a multiple of π, write a formula for the distance between P_1 and P_2.

(b) Use the Law of Cosines to prove that the distance between P_1 and P_2 is given by

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_1 - \theta_2)}$$

(c) Writing to Learn Does the formula in part (b) agree with the formula(s) you found in part (a)? Explain.

62. Watching Your θ-Step Consider the polar curve $r = 4 \sin \theta$. Describe the graph for each of the following.

(a) $0 \leq \theta \leq \pi/2$ **(b)** $0 \leq \theta \leq 3\pi/4$
(c) $0 \leq \theta \leq 3\pi/2$ **(d)** $0 \leq \theta \leq 4\pi$

In Exercises 63–66, use the results of Exercise 61 to find the distance between the points with given polar coordinates.

63. (2, 10°), (5, 130°) ≈ 6.24

64. (4, 20°), (6, 65°) ≈ 4.25

65. (−3, 25°), (−5, 160°) ≈ 7.43

66. (6, −35°), (8, −65°) ≈ 4.11

Extending the Ideas

67. Graphing Polar Equations Parametrically Find parametric equations for the polar curve $r = f(\theta)$.

Group Activity In Exercises 68–71, use what you learned in Exercise 67 to write parametric equations for the given polar equation. Support your answers graphically.

68. $r = 2 \cos \theta$

69. $r = 5 \sin \theta$

70. $r = 2 \sec \theta$

71. $r = 4 \csc \theta$