

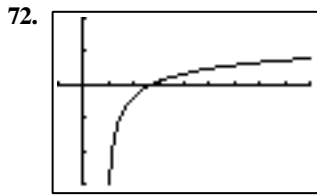
70. Recall that $y = \log_a x$ can be written as $x = a^y$.

$$\begin{aligned} \text{Let } y &= \log_a b \\ a^y &= b \\ \log a^y &= \log b \\ y \log a &= \log b \\ y &= \frac{\log b}{\log a} = \log_a b \end{aligned}$$

71. Let $y = \frac{\log x}{\ln x}$. By the change-of-base formula,

$$y = \frac{\log x}{\log e} = \log x \cdot \frac{\log e}{\log x} = \log e \approx 0.43$$

Thus, y is a constant function.



$[-1, 9]$ by $[-3, 2]$

Domain: $(1, \infty)$

Range: $(-\infty, \infty)$

Continuous

Increasing

Not symmetric

Vertical asymptote: $x = 1$

$\lim_{x \rightarrow \infty} f(x) = \infty$

One-to one, hence invertible ($f^{-1}(x) = e^{e^x}$)

Section 3.5 Equation Solving and Modeling

Exploration 1

- $\log(4 \cdot 10) \approx 1.60206$
 $\log(4 \cdot 10^2) \approx 2.60206$
 $\log(4 \cdot 10^3) \approx 3.60206$
 $\log(4 \cdot 10^4) \approx 4.60206$
 $\log(4 \cdot 10^5) \approx 5.60206$
 $\log(4 \cdot 10^6) \approx 6.60206$
 $\log(4 \cdot 10^7) \approx 7.60206$
 $\log(4 \cdot 10^8) \approx 8.60206$
 $\log(4 \cdot 10^9) \approx 9.60206$
 $\log(4 \cdot 10^{10}) \approx 10.60206$
- The integers increase by 1 for every increase in a power of 10.
- The decimal parts are exactly equal.
- $4 \cdot 10^{10}$ is nine orders of magnitude greater than $4 \cdot 10$.

Quick Review 3.5

In #1–4, graphical support (i.e., graphing both functions on a square window) is also useful.

- $f(g(x)) = e^{2 \ln(x^{1/2})} = e^{\ln x} = x$ and $g(f(x)) = \ln(e^{2x})^{1/2} = \ln(e^x) = x$.
- $f(g(x)) = 10^{(\log x^2)/2} = 10^{\log x} = x$ and $g(f(x)) = \log(10^{x/2})^2 = \log(10^x) = x$.

- $f(g(x)) = \frac{1}{3} \ln(e^{3x}) = \frac{1}{3} (3x) = x$ and $g(f(x)) = e^{3(1/3 \ln x)} = e^{\ln x} = x$.
- $f(g(x)) = 3 \log(10^{x/6})^2 = 6 \log(10^{x/6}) = 6(x/6) = x$ and $g(f(x)) = 10^{(3 \log x^2)/6} = 10^{(\log x^2)/2} = 10^{\log x} = x$.
- 7.783×10^8 km
- 1×10^{-15} m
- 602,000,000,000,000,000,000,000
- 0.000 000 000 000 000 000 000 000 001 66 (26 zeros between the decimal point and the 1)
- $(1.86 \times 10^5)(3.1 \times 10^7) = (1.86)(3.1) \times 10^{5+7} = 5.766 \times 10^{12}$
- $\frac{8 \times 10^{-7}}{5 \times 10^{-6}} = \frac{8}{5} \times 10^{-7-(-6)} = 1.6 \times 10^{-1}$

Section 3.5 Exercises

For #1–18, take a logarithm of both sides of the equation, when appropriate.

- $36 \left(\frac{1}{3}\right)^{x/5} = 4$
 $\left(\frac{1}{3}\right)^{x/5} = \frac{1}{9}$
 $\left(\frac{1}{3}\right)^{x/5} = \left(\frac{1}{3}\right)^2$
 $\frac{x}{5} = 2$
 $x = 10$
- $32 \left(\frac{1}{4}\right)^{x/3} = 2$
 $\left(\frac{1}{4}\right)^{x/3} = \frac{1}{16}$
 $\left(\frac{1}{4}\right)^{x/3} = \left(\frac{1}{4}\right)^2$
 $\frac{x}{3} = 2$
 $x = 6$
- $2 \cdot 5^{x/4} = 250$
 $5^{x/4} = 125$
 $5^{x/4} = 5^3$
 $\frac{x}{4} = 3$
 $x = 12$
- $3 \cdot 4^{x/2} = 96$
 $4^{x/2} = 32$
 $4^{x/2} = 4^{5/2}$
 $\frac{x}{2} = \frac{5}{2}$
 $x = 5$
- $10^{-x/3} = 10$, so $-x/3 = 1$, and therefore $x = -3$.
- $5^{-x/4} = 5$, so $-x/4 = 1$, and therefore $x = -4$.
- $x = 10^4 = 10,000$
- $x = 2^5 = 32$
- $x - 5 = 4^{-1}$, so $x = 5 + 4^{-1} = 5.25$.
- $1 - x = 4^1$, so $x = -3$.

$$11. x = \frac{\ln 4.1}{\ln 1.06} = \log_{1.06} 4.1 \approx 24.2151$$

$$12. x = \frac{\ln 1.6}{\ln 0.98} = \log_{0.98} 1.6 \approx -23.2644$$

$$13. e^{0.035x} = 4, \text{ so } 0.035x = \ln 4, \text{ and therefore}$$

$$x = \frac{1}{0.035} \ln 4 \approx 39.6084.$$

$$14. e^{0.045x} = 3, \text{ so } 0.045x = \ln 3, \text{ and therefore}$$

$$x = \frac{1}{0.045} \ln 3 \approx 24.4136.$$

$$15. e^{-x} = \frac{3}{2}, \text{ so } -x = \ln \frac{3}{2}, \text{ and therefore}$$

$$x = -\ln \frac{3}{2} \approx -0.4055.$$

$$16. e^{-x} = \frac{5}{3}, \text{ so } -x = \ln \frac{5}{3}, \text{ and therefore}$$

$$x = -\ln \frac{5}{3} \approx -0.5108.$$

$$17. \ln(x - 3) = \frac{1}{3}, \text{ so } x - 3 = e^{1/3}, \text{ and therefore}$$

$$x = 3 + e^{1/3} \approx 4.3956.$$

$$18. \log(x + 2) = -2, \text{ so } x + 2 = 10^{-2}, \text{ and therefore}$$

$$x = -2 + 10^{-2} = -1.99.$$

$$19. \text{ We must have } x(x + 1) > 0, \text{ so } x < -1 \text{ or } x > 0.$$

Domain: $(-\infty, -1) \cup (0, \infty)$; graph (e).

$$20. \text{ We must have } x > 0 \text{ and } x + 1 > 0, \text{ so } x > 0.$$

Domain: $(0, \infty)$; graph (f).

$$21. \text{ We must have } \frac{x}{x + 1} > 0, \text{ so } x < -1 \text{ or } x > 0.$$

Domain: $(-\infty, -1) \cup (0, \infty)$; graph (d).

$$22. \text{ We must have } x > 0 \text{ and } x + 1 > 0, \text{ so } x > 0.$$

Domain: $(0, \infty)$; graph (c).

$$23. \text{ We must have } x > 0. \text{ Domain: } (0, \infty); \text{ graph (a).}$$

$$24. \text{ We must have } x^2 > 0, \text{ so } x \neq 0.$$

Domain: $(-\infty, 0) \cup (0, \infty)$; graph (b).

For #25–38, algebraic solutions are shown (and are generally the only way to get *exact* answers). In many cases solving graphically would be faster; graphical support is also useful.

$$25. \text{ Write both sides as powers of 10, leaving } 10^{\log x^2} = 10^6, \text{ or}$$

$$x^2 = 1,000,000. \text{ Then } x = 1000 \text{ or } x = -1000.$$

$$26. \text{ Write both sides as powers of } e, \text{ leaving } e^{\ln x^2} = e^4, \text{ or}$$

$$x^2 = e^4. \text{ Then } x = e^2 \approx 7.389 \text{ or } x = -e^2 \approx -7.389.$$

$$27. \text{ Write both sides as powers of 10, leaving } 10^{\log x^4} = 10^2, \text{ or}$$

$$x^4 = 100. \text{ Then } x^2 = 10, \text{ and } x = \pm\sqrt{10}.$$

$$28. \text{ Write both sides as powers of } e, \text{ leaving } e^{\ln x^6} = e^{12}, \text{ or}$$

$$x^6 = e^{12}. \text{ Then } x^2 = e^4, \text{ and } x = \pm e^2.$$

$$29. \text{ Multiply both sides by } 3 \cdot 2^x, \text{ leaving } (2^x)^2 - 1 = 12 \cdot 2^x,$$

$$\text{ or } (2^x)^2 - 12 \cdot 2^x - 1 = 0. \text{ This is quadratic in } 2^x,$$

$$\text{leading to } 2^x = \frac{12 \pm \sqrt{144 + 4}}{2} = 6 \pm \sqrt{37}. \text{ Only}$$

$6 + \sqrt{37}$ is positive, so the only answer is

$$x = \frac{\ln(6 + \sqrt{37})}{\ln 2} = \log_2(6 + \sqrt{37}) \approx 3.5949.$$

$$30. \text{ Multiply both sides by } 2 \cdot 2^x, \text{ leaving } (2^x)^2 + 1 = 6 \cdot 2^x, \text{ or}$$

$$(2^x)^2 - 6 \cdot 2^x + 1 = 0. \text{ This is quadratic in } 2^x, \text{ leading to}$$

$$2^x = \frac{6 \pm \sqrt{36 - 4}}{2} = 3 \pm 2\sqrt{2}. \text{ Then}$$

$$x = \frac{\ln(3 \pm 2\sqrt{2})}{\ln 2} = \log_2(3 \pm 2\sqrt{2}) \approx \pm 2.5431.$$

$$31. \text{ Multiply both sides by } 2e^x, \text{ leaving } (e^x)^2 + 1 = 8e^x, \text{ or}$$

$$(e^x)^2 - 8e^x + 1 = 0. \text{ This is quadratic in } e^x, \text{ leading to}$$

$$e^x = \frac{8 \pm \sqrt{64 - 4}}{2} = 4 \pm \sqrt{15}. \text{ Then}$$

$$x = \ln(4 \pm \sqrt{15}) \approx \pm 2.0634.$$

$$32. \text{ This is quadratic in } e^x, \text{ leading to}$$

$$e^x = \frac{-5 \pm \sqrt{25 + 24}}{4} = \frac{-5 \pm 7}{4}. \text{ Of these two}$$

numbers, only $\frac{-5 + 7}{4} = \frac{1}{2}$ is positive, so $x = \ln \frac{1}{2}$
 $\approx -0.6931.$

$$33. \frac{500}{200} = 1 + 25e^{0.3x}, \text{ so } e^{0.3x} = \frac{3}{50} = 0.06, \text{ and therefore}$$

$$x = \frac{1}{0.3} \ln 0.06 \approx -9.3780.$$

$$34. \frac{400}{150} = 1 + 95e^{-0.6x}, \text{ so } e^{-0.6x} = \frac{1}{57}, \text{ and therefore}$$

$$x = \frac{1}{-0.6} \ln \frac{1}{57} \approx 6.7384.$$

$$35. \text{ Multiply by 2, then combine the logarithms to obtain}$$

$$\ln \frac{x + 3}{x^2} = 0. \text{ Then } \frac{x + 3}{x^2} = e^0 = 1, \text{ so } x + 3 = x^2.$$

The solutions to this quadratic equation are

$$x = \frac{1 \pm \sqrt{1 + 12}}{2} = \frac{1}{2} \pm \frac{1}{2}\sqrt{13} \approx 2.3028.$$

$$36. \text{ Multiply by 2, then combine the logarithms to obtain}$$

$$\log \frac{x^2}{x + 4} = 2. \text{ Then } \frac{x^2}{x + 4} = 10^2 = 100, \text{ so}$$

$$x^2 = 100(x + 4). \text{ The solutions to this quadratic equation}$$

$$\text{are } x = \frac{100 \pm \sqrt{10000 + 1600}}{2} = 50 \pm 10\sqrt{29}. \text{ The}$$

original equation requires that $x > 0$, so $50 - 10\sqrt{29}$ is extraneous; the only actual solution is
 $x = 50 + 10\sqrt{29} \approx 103.852.$

$$37. \ln[(x - 3)(x + 4)] = 3 \ln 2, \text{ so } (x - 3)(x + 4) = 8, \text{ or}$$

$$x^2 + x - 20 = 0. \text{ This factors to } (x - 4)(x + 5) = 0, \text{ so}$$

$$x = 4 \text{ (an actual solution) or } x = -5 \text{ (extraneous, since}$$

$$x - 3 \text{ and } x + 4 \text{ must be positive).}$$

$$38. \log[(x - 2)(x + 5)] = 2 \log 3, \text{ so } (x - 2)(x + 5) = 9,$$

$$\text{ or } x^2 + 3x - 19 = 0.$$

$$\text{Then } x = \frac{-3 \pm \sqrt{9 + 76}}{2} = -\frac{3}{2} \pm \frac{1}{2}\sqrt{85}. \text{ The actual}$$

solution is $x = -\frac{3}{2} + \frac{1}{2}\sqrt{85} \approx 3.1098$; since $x - 2$ must be positive, the other algebraic solution,

$$x = -\frac{3}{2} - \frac{1}{2}\sqrt{85}, \text{ is extraneous.}$$

$$39. \text{ A \$100 bill has the value of 1000, or } 10^3, \text{ dimes so they}$$

$$\text{differ by an order of magnitude of 3.}$$